

Vector boson production at hadron colliders: a fully exclusive QCD calculation at NNLO

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Abstract

We consider QCD radiative corrections to the production of W and Z bosons in hadron collisions. We present a fully exclusive calculation up to next-to-next-to-leading order (NNLO) in QCD perturbation theory. To perform this NNLO computation, we use a recently proposed version of the subtraction formalism. The calculation includes the γ - Z interference, finite-width effects, the leptonic decay of the vector bosons and the corresponding spin correlations. Our calculation is implemented in a parton level Monte Carlo program. The program allows the user to apply arbitrary kinematical cuts on the final-state leptons and the associated jet activity, and to compute the corresponding distributions in the form of bin histograms. We show selected numerical results at the Tevatron and the LHC.

The production of W and Z bosons in hadron collisions through the Drell–Yan (DY) mechanism [1] is extremely important for physics studies at hadron colliders. These processes have large production rates and offer clean experimental signatures, given the presence of at least one high- p_T lepton in the final state. Studies of the production of W bosons at the Tevatron lead to precise determinations of the W mass and width [2]. The DY process is also expected to provide standard candles for detector calibration during the first stage of the LHC running.

Because of the above reasons, it is essential to have accurate theoretical predictions for the vector-boson production cross sections and the associated distributions. Theoretical predictions with high precisions demand detailed computations of radiative corrections. The QCD corrections to the total cross section [3] and to the rapidity distribution [4] of the vector boson are known up to the next-to-next-to-leading order (NNLO) in the strong coupling α_s . The fully exclusive NNLO calculation, including the leptonic decay of the vector boson, has been completed more recently [5]. Full electroweak corrections at $\mathcal{O}(\alpha)$ have been computed for both W [6] and Z production [7].

In this Letter we present a new computation of the NNLO QCD corrections to vector boson production in hadron collisions. The calculation includes the γ – Z interference, finite-width effects, the leptonic decay of the vector bosons and the corresponding spin correlations. Our calculation parallels the one recently completed for Higgs boson production [8, 9], and it is performed by using the same method.

The evaluation of higher-order QCD corrections to hard-scattering processes is complicated by the presence of infrared (IR) singularities at intermediate stages of the calculation that prevents a straightforward implementation of numerical techniques. Despite this difficulty, general methods have been developed in the last two decades, which allow us to handle and cancel IR singularities [10, 11, 12] appearing in NLO QCD calculations. In the last few years, several research groups have been working on extensions of these methods to NNLO [13, 14, 15, 16, 17], and, recently, the NNLO calculation for $e^+e^- \rightarrow 3$ jets was completed by two groups [18, 19]. Parallely, a new general method [20], based on sector decomposition [21], has been proposed and applied to the NNLO calculations of $e^+e^- \rightarrow 2$ jets [22], Higgs [23] and vector [5] boson production in hadron collisions, and to some decay processes [24]. Our method [8] applies to the production of colourless high-mass systems in hadron collisions, and is based on an extension of the subtraction formalism [11, 12] to NNLO that we briefly recall below.

We consider the inclusive hard-scattering reaction

$$h_1 + h_2 \rightarrow V(q) + X, \quad (1)$$

where the collision of the two hadrons h_1 and h_2 produces the vector boson V ($V = Z/\gamma^*, W^+$ or W^-), with four-momentum q and high invariant mass $\sqrt{q^2}$. At next-to-leading order (NLO), two kinds of corrections contribute: i) *real* corrections, where one parton recoils against V ; ii) *one-loop virtual* corrections to the leading order (LO) subprocess. Both contributions are separately IR divergent, but the divergences cancel in the sum. At NNLO, three kinds of corrections must be considered: i) *double real* contributions, where two partons recoil against V ; ii) *real-virtual* corrections, where one parton recoils against V at one-loop order; iii) *two-loop virtual* corrections to the LO subprocess. The three contributions are still separately divergent, and the calculation has to be organized so as to explicitly achieve the cancellation of the IR divergences.

We first note that, at LO, the transverse momentum q_T of V is exactly zero. As a consequence, as long as $q_T \neq 0$, the (N)NLO contributions are actually given by the (N)LO contributions to

$V + \text{jet(s)}$. Thus, we can write the cross section as

$$d\sigma_{(N)NLO}^V|_{q_T \neq 0} = d\sigma_{(N)LO}^{V+\text{jets}} . \quad (2)$$

This means that, when $q_T \neq 0$, the IR divergences in our NNLO calculation are those in $d\sigma_{NLO}^{V+\text{jets}}$: they can be treated by using available NLO methods to handle and cancel IR singularities (e.g., the general NLO methods in Refs. [10, 11, 12]). The only remaining singularities of NNLO type are associated to the limit $q_T \rightarrow 0$. Following Ref. [8] we treat them by an additional subtraction. Our key point is that the singular behaviour of $d\sigma_{(N)LO}^{V+\text{jets}}$ when $q_T \rightarrow 0$ is well known: it comes out in the resummation program [25] of logarithmically-enhanced contributions to transverse-momentum distributions. Therefore, the additional subtraction can be worked out by using a counterterm, $d\sigma_{(N)LO}^{CT}$, whose general structure [8] depends only on the flavour of the initial-state partons involved in the LO partonic subprocess ($q\bar{q}$ annihilation in the case of vector-boson production, gg fusion in the case of Higgs boson production).

Our extension of Eq. (2) to include the contribution at $q_T = 0$ is [8]:

$$d\sigma_{(N)NLO}^V = \mathcal{H}_{(N)NLO}^V \otimes d\sigma_{LO}^V + \left[d\sigma_{(N)LO}^{V+\text{jets}} - d\sigma_{(N)LO}^{CT} \right] . \quad (3)$$

Comparing with the right-hand side of Eq. (2), we have subtracted the (N)LO counterterm $d\sigma_{(N)LO}^{CT}$ and added a contribution at $q_T = 0$, which is needed to obtain the correct total cross section. The coefficient $\mathcal{H}_{(N)NLO}^V$ does not depend on q_T and is obtained by the (N)NLO truncation of the hard-scattering perturbative function

$$\mathcal{H}^V = 1 + \frac{\alpha_S}{\pi} \mathcal{H}^{V(1)} + \left(\frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{V(2)} + \dots . \quad (4)$$

According to Eq. (3), the NLO calculation of $d\sigma^V$ requires the knowledge of $\mathcal{H}^{V(1)}$ and the LO calculation of $d\sigma^{V+\text{jets}}$. The general (process-independent) form of the coefficient $\mathcal{H}^{(1)}$ is known: the precise relation between $\mathcal{H}^{(1)}$ and the IR finite part of the one-loop correction to a generic LO subprocess is explicitly derived in Ref. [26]. At NNLO, the coefficient $\mathcal{H}^{V(2)}$ is also needed, together with the NLO calculation of $d\sigma^{V+\text{jets}}$. The calculation of the general structure of the coefficients $\mathcal{H}^{(2)}$ is in progress. Meanwhile, by using the available analytical results at $\mathcal{O}(\alpha_S^2)$ for the total cross section [3] and the transverse-momentum spectrum [27] of the vector boson, we have explicitly computed the coefficient $\mathcal{H}^{V(2)}$ of the DY process. Since the NLO corrections $d\sigma_{NLO}^{V+\text{jets}}$ to $q\bar{q} \rightarrow V + \text{jet(s)}$ are also available [28], using Eq. (3) we are able to complete our fully-exclusive NNLO calculation of vector-boson production.

We have encoded our NNLO computation in a parton level Monte Carlo program, in which we can implement arbitrary IR safe cuts on the final-state leptons and the associated jet activity.

In the following we present an illustrative selection of numerical results for Z and W production at the Tevatron and the LHC. We consider u, d, s, c, b quarks in the initial state. In the case of W^\pm production, we use the (unitarity constrained) CKM matrix elements $V_{ud} = 0.97419$, $V_{us} = 0.2257$, $V_{ub} = 0.00359$, $V_{cd} = 0.2256$, $V_{cs} = 0.97334$, $V_{cb} = 0.0415$ from the PDG 2008 [29]. In the case of Z production, additional Feynman diagrams with fermionic triangles should be taken into account. Their contribution cancels out for each isospin multiplet when massless quarks are considered. The effect of a finite top-quark mass in the third generation has been considered and found extremely small [30], so it is neglected in our calculation. As for the electroweak couplings, we use the so

called G_μ scheme, where the input parameters are G_F , m_Z , m_W . In particular we use the values $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$, $m_Z = 91.1876 \text{ GeV}$, $\Gamma_Z = 2.4952 \text{ GeV}$, $m_W = 80.398 \text{ GeV}$ and $\Gamma_W = 2.141 \text{ GeV}$. We use the MSTW2008 [31] sets of parton distributions, with densities and α_S evaluated at each corresponding order (i.e., we use $(n+1)$ -loop α_S at $N^n\text{LO}$, with $n = 0, 1, 2$). The renormalization and factorization scales are fixed to the value $\mu_R = \mu_F = m_V$, where m_V is the mass of the vector boson.

We start the presentation of our results by considering the inclusive production of e^+e^- pairs from the decay of an on-shell Z boson at the LHC. In Fig. 1 (left panel) we show the rapidity distribution of the e^+e^- pair at LO, NLO and NNLO, computed by using the MSTW2008 partons. The corresponding cross sections[†] are $\sigma_{LO} = 1.761 \pm 0.001 \text{ nb}$, $\sigma_{NLO} = 2.030 \pm 0.001 \text{ nb}$ and

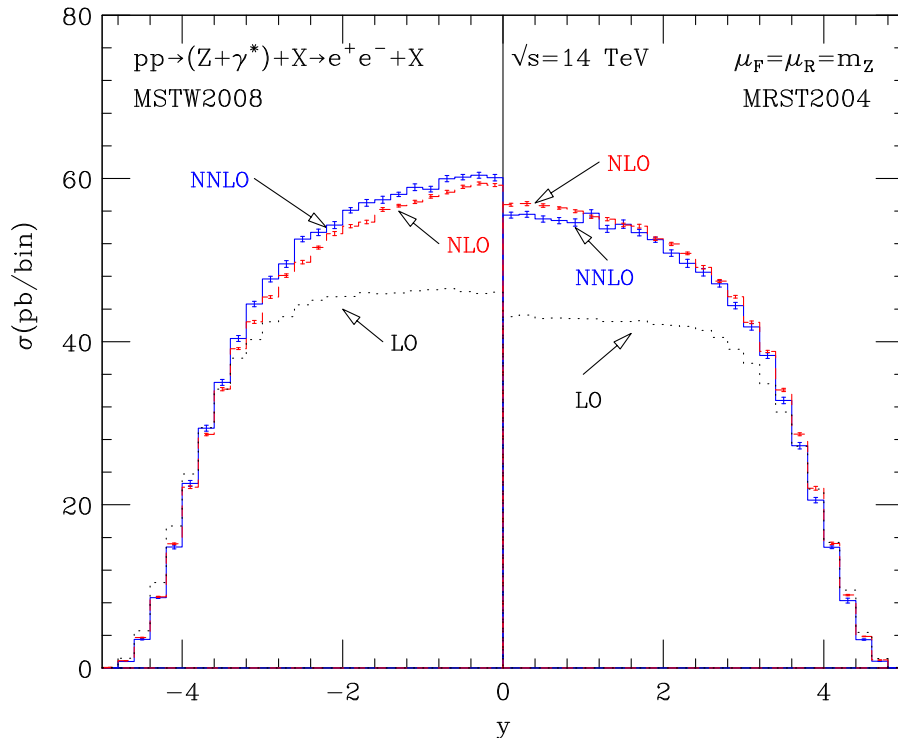


Figure 1: *Rapidity distribution of an on-shell Z boson at the LHC. Results obtained with the MSTW2008 set (left panel) are compared with those obtained with the MRST2004 set (right panel).*

$\sigma_{NNLO} = 2.089 \pm 0.003 \text{ nb}$. The total cross section is increased by about 3% in going from NLO to NNLO. In Fig. 1 (right panel) we also show the results obtained by using the MRST2002 LO [32] and MRST2004 [33] sets of parton distribution functions. The corresponding cross sections are $\sigma_{LO} = 1.629 \pm 0.001 \text{ nb}$, $\sigma_{NLO} = 1.992 \pm 0.001 \text{ nb}$ and $\sigma_{NNLO} = 1.954 \pm 0.003 \text{ nb}$. In this case the total cross section is decreased by about 2% in going from NLO to NNLO.

We next consider the production of e^+e^- pairs from Z/γ^* bosons at the Tevatron. For each event, we classify the lepton transverse momenta according to their minimum and maximum values, $p_{T\min}$ and $p_{T\max}$. The leptons are required to have a minimum p_T of 20 GeV and pseudo-rapidity $|\eta| < 2$. Their invariant mass is required to be in the range $70 \text{ GeV} < m_{e^+e^-} < 110 \text{ GeV}$.

[†]Throughout the paper, the errors on the values of the cross sections and the error bars in the plots refer to an estimate of the numerical errors in the Monte Carlo integration.

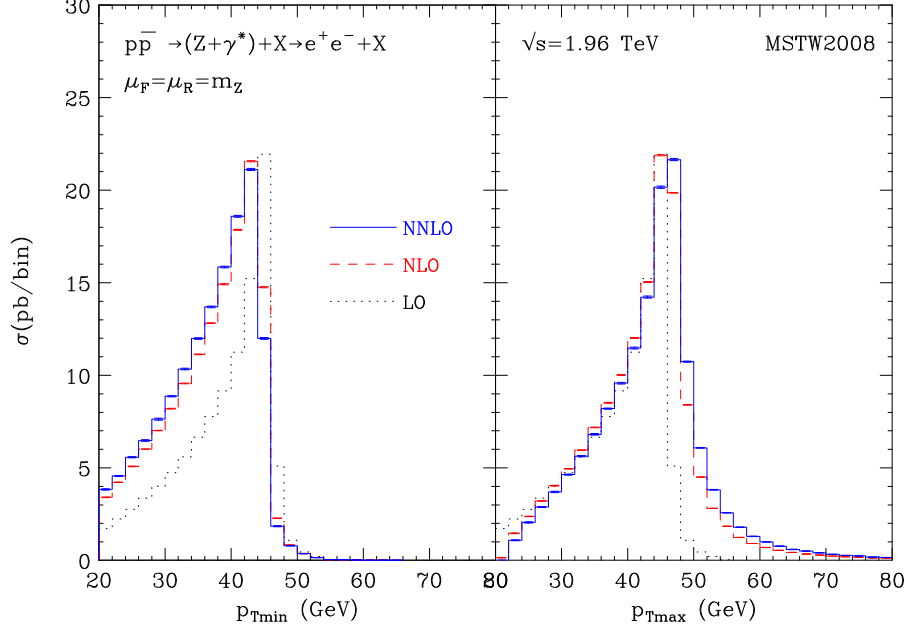


Figure 2: *Distributions in $p_{T\min}$ and $p_{T\max}$ for the Z signal at the Tevatron.*

The accepted cross sections are $\sigma_{LO} = 103.37 \pm 0.04$ pb, $\sigma_{NLO} = 140.43 \pm 0.07$ pb and $\sigma_{NNLO} = 143.86 \pm 0.12$ pb. In Fig. 2 we plot the distributions in $p_{T\min}$ and $p_{T\max}$ at LO, NLO and NNLO. We note that at LO the $p_{T\min}$ and $p_{T\max}$ distributions are kinematically bounded by $p_T \leq Q_{\max}/2$, where $Q_{\max} = 110$ GeV is the maximum allowed invariant mass of the e^+e^- pairs. The NNLO corrections have a visible impact on the shape of the $p_{T\min}$ and $p_{T\max}$ distribution and make the $p_{T\min}$ distribution softer, and the $p_{T\max}$ distribution harder.

We finally consider the production of a charged lepton plus missing p_T through the decay of a W boson ($W = W^+, W^-$) at the Tevatron. The charged lepton is selected to have $p_T > 20$ GeV and $|\eta| < 2$ and the missing p_T of the event should be larger than 25 GeV. We define the transverse mass of the event as $m_T = \sqrt{2p_T^l p_T^{\text{miss}}(1 - \cos \phi)}$, where ϕ is the angle between the the p_T of the lepton and the missing p_T . The accepted cross sections are $\sigma_{LO} = 1.161 \pm 0.001$ nb, $\sigma_{NLO} = 1.550 \pm 0.001$ nb and $\sigma_{NNLO} = 1.586 \pm 0.002$ nb.

In Fig. 3 we show the m_T distribution at LO, NLO and NNLO. We note that at LO the distribution has a kinematical boundary at $m_T = 50$ GeV. This is due to the fact that at LO the W is produced with zero transverse momentum: therefore, the requirement $p_T^{\text{miss}} > 25$ GeV sets $m_T \geq 50$ GeV. Around the region where $m_T = 50$ GeV there are perturbative instabilities in going from LO to NLO and to NNLO. The origin of these perturbative instabilities is well known [34]: since the LO spectrum is kinematically bounded by $m_T \geq 50$ GeV, each higher-order perturbative contribution produces (integrable) logarithmic singularities in the vicinity of the boundary. We also note that, below the boundary, the NNLO corrections to the NLO result are large; for example, they are about +40% at $m_T \sim 30$ GeV. This is not unexpected, since in this region of transverse masses, the $\mathcal{O}(\alpha_s)$ result corresponds to the calculation at the first perturbative order and, therefore, our $\mathcal{O}(\alpha_s^2)$ result is actually only a calculation at the NLO level of perturbative accuracy.

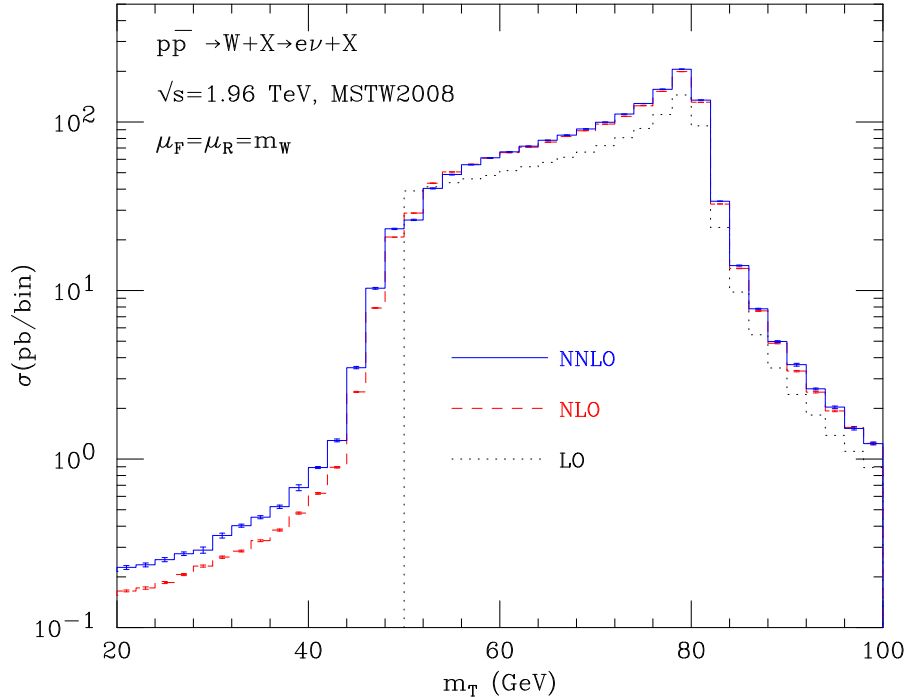


Figure 3: *Transverse mass distribution for W production at the Tevatron.*

We have illustrated a calculation of the cross section for W and Z boson production up to NNLO in QCD perturbation theory. An analogous computation was presented in Ref. [5]. Our calculation is performed with a completely independent method. In the quantitative studies that we have carried out, the two computations give results in numerical agreement. Our calculation is directly implemented in a parton level event generator. This feature makes it particularly suitable for practical applications to the computation of distributions in the form of bin histograms. A public version of our program will be available in the near future.

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